The Effect of the Metric Behavior at Spatial Infinity on Wave Decay in the Presence of Asymptotically Flat Stationary Spacetimes

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Talk Outline

1. Results and Research

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2. Overview of Tataru's Argument

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3. The role of the metric behavior at spatial infinity

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3. The role of the metric behavior at spatial infinity
4. Research Status and Miscellany

Zero Resolvent Expansion in Powers of $\langle r \rangle^{-1}$

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(b) For $\|\langle r \rangle^2 g\| \lesssim 1$:

$$R_0 g = c\langle r \rangle^{-1} + d(r) \cdot \nabla \langle r \rangle^{-1} + e(r)\langle r \rangle^{-2} + v_2$$

Error Term Calculation for f

$$
\begin{aligned}
&P_\tau(R_\tau f - R_0 f e^{-i\tau\langle r\rangle}) \\
&\quad = \left(f - f e^{-i\tau\langle r\rangle}\right) + 2\tau\left(\partial_r + \frac{1}{r}\right)(R_0 f) e^{-i\tau\langle r\rangle} \\
&\qquad + \left(\tau\mathcal{O}(r^{-2-\epsilon}) + \tau^2\mathcal{O}(r^{-1-\epsilon}) + \tau\mathcal{O}(r^{-1-\epsilon})\nabla\right)(R_0 f) e^{-i\tau\langle r\rangle}
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&\quad = \chi_{>|\tau|^{-1}} f + \tau h
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&= \chi_{|\tau|^{-1}} g + \tau^2 h + \tau P_\tau R_0 f e^{-i\tau\langle r\rangle} + \tau P_\tau e_0(r,\tau) e^{-i\tau\langle r\rangle}
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R_0 g = c\langle r\rangle^{-1} + d(r)\cdot\nabla\langle r\rangle^{-1} + e(r)\langle r\rangle^{-2} + v_2
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Resolvent Expansion for Small τ

(a)

$$R_\tau f = (R_0 f)e^{-i\tau\langle r\rangle} + R_\tau \left(\chi_{>|\tau|^{-1}}(r)f + \tau h\right)$$

where

$$h \sim r^{-2-\epsilon} e^{i\tau\langle r\rangle}$$

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(b)

$$R_\tau g = \left(R_0 g + \tau R_0 \tilde{g} + \tau e_0(r,\tau)\right)e^{-i\tau\langle r\rangle} + R_\tau\left(\chi_{>|\tau|^{-1}}(r)g + \tau^2 h\right)$$

where

$$\|\langle r\rangle\tilde{g}\| \lesssim 1 \qquad e_0 \sim \log(\min(r,|\tau|^{-1})) \qquad h \sim r^{-2-\epsilon}e^{i\tau\langle r\rangle}$$

Decay Rates

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$$|u_c| \lesssim \langle r \rangle^{-1} \langle t - r \rangle^{-N}$$

$$|u_d| \lesssim \langle t + r \rangle^{-3}$$

Conclusion

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5. Other questions

Thank you for your attention!